The relativistic field theory model of the deuteron from low–energy QCD

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Abstract

The relativistic field theory model of the deuteron (RFMD) is reformulated from the first principles of QCD. The deuteron appears as a neutron–proton collective excitation, i.e. a Cooper np–pair, induced by a phenomenological local four–nucleon interaction in the nuclear phase of QCD. The RFMD describes the deuteron coupled to hadrons through one–nucleon loop exchanges providing a minimal transfer of nucleon flavours from initial to final nuclear states and accounting for contributions of nucleon–loop anomalies which are completely determined by one–nucleon loop diagrams. The dominance of contributions of nucleon–loop anomalies to effective Lagrangians of low–energy nuclear interactions is justified in the large N_C expansion, where N_C is the number of quark colours.

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1 Introduction

In our recent publications [1–5] we have formulated the relativistic field theory model of the deuteron (RFMD) [1,2]. The first version of the RFMD [1,2] formulated in analogy with a phenomenological σ–model had no obvious connection with QCD [2]. Unlike the potential model approach (PMA) [6–8] and the Effective Field Theory (EFT) approach [9–14] the RFMD takes into account non–perturbative contributions of high–energy (or short–distance) fluctuations of virtual nucleon (N) and anti–nucleon ($\bar{N}$) fields, i.e. $N\bar{N}$ fluctuations, in the form of anomalies of one–nucleon loop diagrams[1]. The description of one–nucleon loop anomalies caused by contributions of high–energy (or short–distance) fluctuations of virtual nucleon fields goes beyond the scope of both the PMA and the EFT approach. This is due to the absence in these approaches anti–nucleon degrees of freedom related to the non–perturbative quantum vacuum – the nucleon Dirac sea [15].

It is known that the nucleon Dirac sea cannot be ignored fully in the low–energy nuclear physics. For example, high–energy $N\bar{N}$ fluctuations of the nucleon Dirac sea polarized by the nuclear medium decrease the scalar nuclear density in the nuclear interior of finite nuclei by 15% [16]. This effect has been obtained within quantum field theoretic approaches in terms of one-nucleon loop exchanges. Unfortunately, contributions of nucleon–loop anomalies have not been taken into account in these approaches. The RFMD allows to fill this blank. In fact, in accord the analysis carried out in Refs. [17] nucleon–loop anomalies can be interpreted as non–trivial contributions of the nucleon Dirac sea.

In this paper we change the starting ideas of the RFMD having been formulated in [1,2]. We show that the RFMD is fully motivated by QCD and describes low–energy nuclear interactions in the nuclear phase of QCD through one–nucleon loop exchanges. Within the large N_C expansion [18,19] we justify the dominance of one–nucleon loop anomaly contributions to effective Lagrangians describing the deuteron itself and processes of low–energy interactions of the deuteron coupled to other particles.

[1] In Ref.[5] we have considered a modified version of the RFMD which is not well defined due to a violation of Lorentz invariance of the effective four–nucleon interaction describing N + N → N + N transitions. This violation has turned out to be incompatible with a dominance of one–nucleon loop anomalies which are Lorentz covariant. Thereby, the astrophysical factor for the solar proton burning calculated in Ref.[5] and enhanced by a factor of 1.4 with respect to the recommended value [7] is not good established.

The paper is organized as follows. In Sect. 2 we discuss non–perturbative phases of QCD and formulate the RFMD from the first principles of QCD. In Sect. 3 we derive the effective Lagrangian for the deuteron field induced in the nuclear phase of QCD as the neutron–proton collective excitation (the Cooper np–pair) by a phenomenological local four–nucleon interaction. We demonstrate the dominant role of one–nucleon loop anomalies for the formation of the effective Lagrangian of the physical deuteron field. In Sect. 4 we investigate the electromagnetic properties of the deuteron and derive the effective Lagrangian of the deuteron field, the Cooper np–pair, coupled to an external electromagnetic field describing the magnetic dipole and electric quadrupole moments of the physical deuteron. In the Conclusion we discuss the obtained results.

2 Non–perturbative phases of QCD

The derivation of the RFMD from the first principles of QCD goes through three non–perturbative phases of the quark–gluon system. We call them as: 1) the low–energy quark–gluon phase (low–energy QCD), 2) the hadronic phase and 3) the nuclear phase.

The low–energy quark–gluon phase of QCD can be obtained by integrating over fluctuations of quark and gluon fields at energies above the scale of spontaneous breaking of chiral symmetry (SBχS) $\Lambda_{\chi} \simeq 1\,\mathrm{GeV}$. This results in an effective field theory, low–energy QCD, describing strong low–energy interactions of quarks and gluons. The low–energy quark–gluon phase of QCD characterizes itself by the appearance of low–energy gluon–field configurations leading to electric colour fluxes responsible for formation of a linearly rising interquark potential. The former realizes quark confinement.

For the transition to *the hadronic phase of QCD* one should, first, integrate out low–energy gluon degrees of freedom. Integrating over gluon degrees of freedom fluctuating around low–energy gluon–field configurations responsible for formation of a linearly rising interquark potential one arrives at an effective field theory containing only quark (q) and anti–quark ($\bar{q}$) degrees of freedom. This effective field theory describes strong interactions at energies below the SBχS scale $\Lambda_{\chi} \simeq 1\,\mathrm{GeV}$. The resultant quark system possesses both a chirally invariant and a chirally broken phase. In the chirally invariant phase the effective Lagrangian of the quark system is invariant under chiral $U(3) \times U(3)$ group. The chirally invariant phase of the quark system

is unstable and the transition to the chirally broken phase is advantageous. The chirally broken phase characterizes itself by three non–perturbative phenomena: SBχS, hadronization (creation of bound quark states with quantum numbers of mesons $q\bar{q}$, $qq\bar{q}\bar{q}$, baryons qqq and so on) and confinement. The transition to the chirally broken phase caused by SBχS accompanies itself with hadronization. Due to quark confinement all observed bound quark states should be colourless. As in such an effective field theory gluon degrees of freedom are integrated out, the entire variety of strong low–energy interactions of hadrons at energies below the SBχS scale $\Lambda_\chi \simeq 1\,\mathrm{GeV}$ is described by quark–loop exchanges.

Since nowadays in continuum space–time formulation of QCD the integration over low–energy gluon–field configurations can be hardly performed explicitly, phenomenological approximations of this integration represented by different effective quark models with chiral $U(3) \times U(3)$ symmetry motivated by QCD are welcomed.

The most interesting effective quark model allowing to describe analytically both SBχS and bosonization (creation of bound $q\bar{q}$ states with quantum numbers of observed low–lying mesons) is the extended Nambu–Jona–Lasinio (ENJL) model [20–24] with linear [20,22] and non–linear [21,23] realization of chiral $U(3) \times U(3)$ symmetry. As has been shown in Ref. [24] the ENJL model is fully motivated by low–energy QCD with a linearly rising interquark potential and N_C quark colour degrees of freedom at $N_C \to \infty$. In the ENJL model mesons are described as $q\bar{q}$ collective excitations (the Cooper $q\bar{q}$–pairs) induced by phenomenological local four–quark interactions. Through one–constituent quark–loop exchanges the Copper $q\bar{q}$–pairs acquire the properties of the observed low–lying mesons such as $\pi(140)$, $K(490)$, $\eta(550)$, $\rho(770)$, $\omega(780)$, $K^*(890)$ and so on. For the description of low–lying octet and decuplet of baryons the ENJL model has been extended by the inclusion of local six–quark interactions responsible for creation of baryons as qqq collective excitations [25].

Integrating then out low–energy quark–field fluctuations, that can be performed in terms of constituent quark–loop exchanges, one arrives at *the hadronic phase of QCD* containing only local meson and baryon fields. The couplings of low–lying mesons and baryons are described by Effective Chiral Lagrangians with chiral $U(3) \times U(3)$ symmetry [20–27].

The nuclear phase of QCD characterizes itself by the appearance of bound nucleon states – nuclei. In order to arrive at *the nuclear phase of QCD* we suggest to start with the hadronic phase of QCD and integrate out heavy hadron

degrees of freedom, i.e. all heavy baryon degrees of freedom with masses heavier than masses of low–lying octet and decuplet of baryons and heavy meson degrees of freedom with masses heavier than the SBχS scale $\Lambda_\chi \simeq 1\,\mathrm{GeV}$. At low energies the result of the integration over these heavy hadron degrees of freedom can be represented in the form of phenomenological local many–nucleon interactions. Following the scenario of the hadronic phase of QCD, where hadrons are produced by phenomenological local many–quark interactions as many–quark collective excitations, one can assume that some of these many–nucleon interactions are responsible for creation of many–nucleon collective excitations. These excitations acquire the properties of observed bound nucleon states – nuclei through nucleon–loop and low–lying meson exchanges. This results in an effective field theory describing nuclei and their low–energy interactions in analogy with Effective Chiral Lagrangian approaches [26,27]. Note that Chiral perturbation theory can be naturally incorporated into this effective field theory of low–energy interactions of nuclei.

We would like to emphasize that in this scenario of the quantum field theoretic formation of nuclei and their low–energy interactions nuclei are considered as elementary particles. For the first time, the representation of nuclei as elementary particles has been suggested by Sakita and Goebal [28] and Kim and Primakoff [29] for the description of electromagnetic and weak nuclear processes. We develop the quantum field theoretic approach to the interpretation of nuclei as elementary particles by starting with QCD.

Following this scenario the deuteron, being the lightest bound nucleon state, appears in the nuclear phase of QCD as the neutron–proton collective excitation (the Cooper np–pair) induced by a phenomenological local four–nucleon interaction. Through one–nucleon loop exchanges [1–5,20–24] the Cooper np–pair with quantum numbers of the physical deuteron acquires the properties of the physical deuteron (i) the binding energy $\varepsilon_{\mathrm{D}} = 2.225\,\mathrm{MeV}$, (ii) the electric quadrupole moment $Q_{\mathrm{D}} = 0.286\,\mathrm{fm}^2$ [30] and so on.

3 The deuteron as a Cooper np–pair

In order to describe the deuteron induced as the Cooper np–pair we introduce a phenomenological local four–nucleon interaction caused by the

integration over heavy hadron degrees of freedom

$$\mathcal{L}_{\rm int}(x) = -\frac{g^2_{\rm V}}{4M^2_{\rm N}}\, j^{\dagger}_{\mu}(x) j^{\mu}(x), \tag{3.1}$$

where $g_{\rm V}$ is the phenomenological coupling constant of the RFMD [1–4], $M_{\rm N} = 940\,{\rm MeV}$ is the nucleon mass. We neglect here the electromagnetic mass difference for the neutron and the proton. As has been found in [1,2] the coupling constant $g_{\rm V}$ is related to the electric quadrupole moment of the deuteron $Q_{\rm D}$: $g^2_{\rm V} = 2\pi^2 Q_{\rm D} M^2_{\rm N}$ [2].

The baryon current $j^{\mu}(x)$ with the quantum numbers of the deuteron is defined by [1–4]

$$j^{\mu}(x) = -i\,[\bar{p^c}(x)\gamma^{\mu} n(x) - \bar{n^c}(x)\gamma^{\mu} p(x)]. \tag{3.2}$$

Here $p(x)$ and $n(x)$ are the interpolating fields of the proton and the neutron, $N^c(x) = C\,\bar{N}^T(x)$ and $\bar{N}^c(x) = N^T(x)\,C$, where C is a charge conjugation matrix and T is a transposition. In terms of the electric quadrupole moment of the deuteron the phenomenological local four–nucleon interaction Eq.(3.1) reads

$$\mathcal{L}_{\rm int}(x) = -\frac{1}{2}\,\pi^2\, Q_{\rm D}\, j^{\dagger}_{\mu}(x) j^{\mu}(x). \tag{3.3}$$

Now let us discuss the behaviour of the phenomenological coupling constant $g^2_{\rm V}/4M^2_{\rm N}$ from the point of view of the large N_C expansion in QCD with the $SU(N_C)$ gauge group at $N_C \to \infty$ [18,19]. Suppose, for simplicity, that the phenomenological four–nucleon interaction Eq.(3.1) is caused by exchanges of the scalar $f_0(980)$ and $a_0(980)$ mesons being the lightest states among heavy hadrons we have integrated out.

Through a linear realization of chiral $U(3) \times U(3)$ symmetry and the Goldberger–Treiman relation one can find that the coupling constant of σ–mesons $g_{\sigma{\rm NN}}$, the $q\bar{q}$–scalar mesons, coupled to the octet of low–lying baryons should be of order $g_{\sigma{\rm NN}} \sim O(\sqrt{N_C})$ at $N_C \to \infty$. The scalar mesons $f_0(980)$ and $a_0(980)$ are most likely four–quark states with $qq\bar{q}\bar{q}$ quark structure [31,32]. In the limit $N_C \to \infty$ such $qq\bar{q}\bar{q}$ states are suppressed by a factor $1/N_C$ [19]. Thus, an effective coupling constant of low–energy NN interaction caused by the $qq\bar{q}\bar{q}$ scalar meson exchanges should be of order $O(1/N_C)$ at $N_C \to \infty$. By taking into account that in QCD with $N_C \to \infty$ the nucleon mass $M_{\rm N}$ is proportional to N_C [19], $M_{\rm N} = N_C M_q$, where $M_q \sim 300\,{\rm MeV}$ is

the constituent quark mass, we can introduce the nucleon mass $M_{\rm N}$ in the effective coupling constant as a dimensional parameter absorbing the factor N^2_C, i.e. $g^2_{\rm V}/4M^2_{\rm N}$. This is also required by the correct dependence of the deuteron mass on N_C. As a result the phenomenological coupling constant $g_{\rm V}$ turns out to be of order $O(\sqrt{N_C})$ at $N_C \to \infty$.

We should emphasize that one does not need to know too much about quark structure of heavy hadron degrees of freedom we have integrated out. Without loss of generality one can argue that among the multitude of contributions caused by the integration over heavy hadron degrees of freedom one can always find the required local four–nucleon interaction the effective coupling constant of which behaves like $O(1/N_C)$ at $N_C \to \infty$. As we show below this behaviour of the coupling constant of the phenomenological four–nucleon interaction is consistent with the large N_C dependence of low–energy parameters of the physical deuteron.

The effective Lagrangian of the np–system unstable under creation of the Cooper np–pair with quantum numbers of the deuteron is then defined

$$\begin{aligned}{\cal L}^{\rm np}(x) &= \bar{n}(x)\,(i\gamma^{\mu}\partial_{\mu} - M_{\rm N})\,n(x) + \bar{p}(x)\,(i\gamma^{\mu}\partial_{\mu} - M_{\rm N})\,p(x)\\ &\quad -\frac{g^2_{\rm V}}{4M^2_{\rm N}}\,j^{\dagger}_{\mu}(x)j^{\mu}(x) + \ldots,\end{aligned}\tag{3.4}$$

where ellipses stand for low–energy interactions the neutron and the proton with other fields.

In order to introduce the interpolating local deuteron field we should linearalize the Lagrangian Eq.(3.4). Following the procedure described in Refs. [20–23] for the inclusion of local interpolating meson fields in the ENJL model we get

$$\begin{aligned}&{\cal L}^{\rm np}(x) \to \bar{n}(x)\,(i\gamma^{\mu}\partial_{\mu} - M_{\rm N})\,n(x) + \bar{p}(x)\,(i\gamma^{\mu}\partial_{\mu} - M_{\rm N})\,p(x)\\ &+M^2_0\,D^{\dagger}_{\mu}(x)D^{\mu}(x) + g_{\rm V}j^{\dagger}_{\mu}(x)D^{\mu}(x) + g_{\rm V}j^{\mu}(x)D^{\dagger}_{\mu}(x) + \ldots,\end{aligned}\tag{3.5}$$

where $M_0 = 2\,M_{\rm N}$ and $D^{\mu}(x)$ is a local interpolating field with quantum numbers of the deuteron.

In order to derive the effective Lagrangian of the physical deuteron field we should integrate over nucleon fields in the one–nucleon loop approximation [1–5,20–24]. The one–nucleon loop approximation of low–energy nuclear forces allows (i) to transfer nucleon flavours from an initial to a final nuclear state by a minimal way and (ii) to take into account contributions of nucleon–loop anomalies [1–5,17, 33–35], which are fully defined by one–nucleon loop

diagrams [33–35]. It is well–known that quark–loop anomalies play an important role for the correct description of strong low–energy interactions of low–lying hadrons [20–27]. We argue the dominant role of nucleon–loop anomalies for the correct description of low–energy nuclear forces in nuclear physics. We demonstrate below the dominant role of nucleon–loop anomalies by example of the evaluation of the effective Lagrangian of the free deuteron field.

The effective Lagrangian of the free deuteron field evaluated in the one–nucleon loop approximation is defined by [1–4]:

$$\int d^4x\,\mathcal{L}_{\rm eff}(x) = \int d^4x\,M^2_0\,D^{\dagger}_{\mu}(x)D^{\mu}(x)$$
$$-\int d^4x\int\frac{d^4x_1d^4k_1}{(2\pi)^4}\,e^{\textstyle -ik_1\cdot(x-x_1)}\,D^{\dagger}_{\mu}(x)D_{\nu}(x_1)\,\frac{g^2_{\rm V}}{4\pi^2}\,\Pi^{\mu\nu}(k_1;Q), \quad (3.6)$$

where the structure function $\Pi^{\mu\nu}(k_1;Q)$ is given by

$$\Pi^{\mu\nu}(k_1;Q) = \int\frac{d^4k}{\pi^2i}{\rm tr}\Big\{\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1}\gamma^{\mu}\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}}\gamma^{\nu}\Big\}. \quad (3.7)$$

The 4–momentum $Q = a\,k_1$ is an arbitrary shift of momenta of virtual nucleon fields with an arbitrary parameter a [1,2]. For the evaluation of the Q–dependence of the structure function $\Pi^{\mu\nu}(k_1;Q)$ we apply the procedure suggested by Gertsein and Jackiw [32] (see also [2]):

$$\Pi^{\mu\nu}(k_1;Q) - \Pi^{\mu\nu}(k_1;0) = \int^1_0 dx\,\frac{d}{dx}\,\Pi^{\mu\nu}(k_1;xQ) =$$
$$= \int^1_0 dx\int\frac{d^4k}{\pi^2i}Q^{\lambda}\frac{\partial}{\partial k^{\lambda}}{\rm tr}\Big\{\frac{1}{M_{\rm N}-\hat{k}-x\hat{Q}-\hat{k}_1}\gamma^{\mu}\frac{1}{M_{\rm N}-\hat{k}-x\hat{Q}}\gamma^{\nu}\Big\} =$$
$$= 2\int^1_0 dx\,\lim_{k\to\infty}\Big\langle\frac{Q\cdot k}{k^2}{\rm tr}\{(M_{\rm N}+\hat{k}+x\hat{Q}+\hat{k}_1)\gamma^{\mu}(M_{\rm N}+\hat{k}+x\hat{Q})\gamma^{\nu}\}\Big\rangle =$$
$$= 2\,(2Q^{\mu}Q^{\nu} - Q^2\,g^{\mu\nu}) + 2(k^{\mu}_1Q^{\nu} + k^{\nu}_1Q^{\mu} - k_1\cdot Q\,g^{\mu\nu}) =$$
$$= -2\,a(a+1)\,(k^2_1\,g^{\mu\nu} - 2\,k^{\mu}_1k^{\nu}_1). \quad (3.8)$$

Thus, we obtain

$$\Pi^{\mu\nu}(k_1;Q) - \Pi^{\mu\nu}(k_1;0) = -2\,a(a+1)\,(k^2_1\,g^{\mu\nu} - 2\,k^{\mu}_1k^{\nu}_1). \quad (3.9)$$

We would like to emphasize that the r.h.s. of Eq.(3.9) is an explicit expression completely defined by high–energy (short–distance) $N\bar{N}$ fluctuations, since the virtual momentum k is taken at the limit $k \to \infty$, and related to the anomaly of the one–nucleon loop diagram with two vector vertices (the VV–diagram) [33,35].

The structure function $\Pi^{\mu\nu}(k_1;0)$ has been evaluated in Refs. [1,2,5] and reads

$$\Pi^{\mu\nu}(k_1;0) = \frac{4}{3}(k^2_1 g^{\mu\nu} - k^{\mu}_1 k^{\nu}_1)J_2(M_{\rm N}) + 2g^{\mu\nu}[J_1(M_{\rm N}) + M^2_{\rm N} J_2(M_{\rm N})], \quad (3.10)$$

where $J_1(M_{\rm N})$ and $J_2(M_{\rm N})$ are the quadratically and logarithmically divergent integrals [1,2,5]:

$$\begin{aligned} J_1(M_{\rm N}) &= \int \frac{d^4k}{\pi^2 i}\frac{1}{M^2_{\rm N} - k^2} = 4\int\limits^{\Lambda_{\rm D}}_0 \frac{d|\vec{k}\,|\vec{k}^{\,2}}{(M^2_{\rm N} + \vec{k}^{\,2})^{1/2}},\\ J_2(M_{\rm N}) &= \int \frac{d^4k}{\pi^2 i}\frac{1}{(M^2_{\rm N} - k^2)^2} = 2\int\limits^{\Lambda_{\rm D}}_0 \frac{d|\vec{k}\,|\vec{k}^{\,2}}{(M^2_{\rm N} + \vec{k}^{\,2})^{3/2}}. \end{aligned} \quad (3.11)$$

The cut–off $\Lambda_{\rm D}$ restricts from above 3–momenta of low–energy fluctuations of virtual neutron and proton fields forming the physical deuteron [1–5]. Since in the RFMD the cut–off $\Lambda_{\rm D}$ is much less than the mass of the nucleon, $M_{\rm N} \gg \Lambda_{\rm D}$ [1,2], we use below the relation between the divergent integrals:

$$J_1(M_{\rm N}) = 2\,M^2_{\rm N}\,J_2(M_{\rm N}) = \frac{4}{3}\,\frac{\Lambda^3_{\rm D}}{M_{\rm N}} \sim O(1/N_C). \quad (3.12)$$

Note that in Eq.(3.10) we have taken into account only the leading terms in the external momentum expansion, i.e. the k_1–expansion.

The justification of the dominance of the leading order contributions in expansion in powers of external momenta can be provided in the large N_C expansion. Indeed, in QCD with the $SU(N_C)$ gauge group at $N_C \to \infty$ the baryon mass is proportional to the number of quark colours [19]: $M_{\rm N} \sim N_C$. Since for the derivation of effective Lagrangians describing the deuteron and amplitudes of processes of low–energy interactions of the deuteron coupled to other particles all external momenta of interacting particles should be kept off–mass shell, the masses of virtual nucleon fields taken at $N_C \to \infty$ are larger compared with external momenta. By expanding one–nucleon loop diagrams in powers of $1/M_{\rm N}$ we get an expansion in powers of $1/N_C$. Keeping

the leading order in the large N_C expansion we are leaving with the leading order contributions in an external momentum expansion. We should emphasize that anomalous contributions of one–nucleon loop diagrams are defined by the least powers of an external momentum expansion. This implies that in the RFMD effective Lagrangians of low–energy interactions are completely determined by contributions of one–nucleon loop anomalies. The divergent contributions having the same order in momentum expansion are negligible small compared with the anomalous ones due to the inequality $M_{\rm N} \gg \Lambda_{\rm D}$ [1–5] and the limit $N_C \to \infty$. This justifies the application of the approximation by the leading powers in an external momentum expansion to the evaluation of the effective Lagrangians of the deuteron coupled to itself and an external electromagnetic field [2], and the effective Lagrangians describing amplitudes of low–energy nuclear processes like the solar proton burning p + p $\to$ D + $\rm e^+$ + $\nu_{\rm e}$ and so [2,4].

Collecting all pieces we get the structure function $\Pi^{\mu\nu}(k_1;Q)$ in the form

$$\begin{aligned}\Pi^{\mu\nu}(k_1;Q) &= \frac{4}{3}(k^2_1 g^{\mu\nu} - k^{\mu}_1 k^{\nu}_1) J_2(M_{\rm N}) + 2g^{\mu\nu}[J_1(M_{\rm N}) + M^2_{\rm N} J_2(M_{\rm N})] \\ &\quad -2\,a(a+1)\,(k^2_1\,g^{\mu\nu} - 2\,k^{\mu}_1 k^{\nu}_1). \end{aligned}\tag{3.13}$$

The effective Lagrangian of the free deuteron field is then defined

$$\begin{aligned}{\cal L}_{\rm eff}(x) &= -\frac{1}{2}\Bigg(-\frac{g^2_{\rm V}}{2\pi^2}\,a(a+1) + \frac{g^2_{\rm V}}{3\pi^2}\,J_2(M_{\rm N})\Bigg)\,D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x) \\ &+\Bigg(M^2_0 - \frac{g^2_{\rm V}}{2\pi^2}\,[J_1(M_{\rm N}) + M^2_{\rm N} J_2(M_{\rm N})]\Bigg)\,D^{\dagger}_{\mu}(x)D^{\mu}(x),\end{aligned}\tag{3.14}$$

where $D^{\mu\nu}(x) = \partial^{\mu}D^{\nu}(x) - \partial^{\nu}D^{\mu}(x)$. We have dropped some contributions proportional to the total divergence of the deuteron field, since $\partial_{\mu}D^{\mu}(x) = 0$. For the derivation of Eq.(3.14) we have used the relation

$$\begin{aligned}&\int d^4x \int \frac{d^4x_1 d^4k_1}{(2\pi)^4}\,e^{-ik_1\,\cdot\,(x - x_1)}\,D^{\dagger}_{\mu}(x)D_{\nu}(x_1)(k^2_1 g^{\mu\nu} - k^{\mu}_1 k^{\nu}_1) = \\ &= \frac{1}{2}\int d^4x\,D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x).\end{aligned}\tag{3.15}$$

In order to get a correct kinetic term of the free deuteron field in the effective Lagrangian Eq.(3.14) we should set [2]

$$-\frac{g^2_{\rm V}}{2\pi^2}\,a(a+1) = 1.\tag{3.16}$$

Since a is an arbitrary real parameter, the relation Eq.(3.16) is valid in the case of the existence of real roots. For the existence of real roots of Eq.(3.16) the coupling constant $g_{\rm V}$ should obey the constraint $g^2_{\rm V} > 8\pi^2$ that is satisfied by the numerical value $g_{\rm V} = 11.319$ calculated at $N_C = 3$ [2]. Since $g_{\rm V} \sim O(\sqrt{N_C})$ at $N_C \to \infty$, Eq.(3.16) has real solutions for any $N_C \geq 3$.

Due to Eq.(3.16) the effective Lagrangian of the free deuteron field takes the form

$$\mathcal{L}_{\rm eff}(x) = -\frac{1}{2}\Bigg(1 + \frac{g^2_{\rm V}}{3\pi^2}\,J_2(M_{\rm N})\Bigg)\,D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x)$$
$$+\Bigg(M^2_0 - \frac{g^2_{\rm V}}{2\pi^2}\,[J_1(M_{\rm N}) + M^2_{\rm N}J_2(M_{\rm N})]\Bigg)\,D^{\dagger}_{\mu}(x)D^{\mu}(x). \qquad (3.17)$$

By performing the renormalization of the wave function of the deuteron field [1,2]

$$\Bigg(1 + \frac{g^2_{\rm V}}{3\pi^2}\,J_2(M_{\rm N})\Bigg)^{1/2}\,D^{\mu}(x) \to D^{\mu}(x) \qquad (3.18)$$

and taking into account that $M_{\rm N} \gg \Lambda_{\rm D}$ we arrive at the effective Lagrangian of the free physical deuteron field

$$\mathcal{L}_{\rm eff}(x) = -\frac{1}{2}\,D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x) + M^2_{\rm D}\,D^{\dagger}_{\mu}(x)D^{\mu}(x), \qquad (3.19)$$

where $M_{\rm D} = M_0 - \varepsilon_{\rm D}$ is the mass of the physical deuteron field. The binding energy of the deuteron $\varepsilon_{\rm D}$ reads

$$\varepsilon_{\rm D} = \frac{17}{48}\,\frac{g^2_{\rm V}}{\pi^2}\,\frac{J_1(M_{\rm N})}{M_{\rm N}} = \frac{17}{18}\,Q_{\rm D}\,\Lambda^3_{\rm D} \sim O(1/N_C). \qquad (3.20)$$

We have used here the relation between divergent integrals Eq.(3.11) and expressed the phenomenological coupling constant $g_{\rm V}$ in terms of the electric quadrupole moment of the deuteron $g^2_{\rm V} = 2\pi^2 Q_{\rm D} M^2_{\rm N}$.

At $N_C \to \infty$ the binding energy of the deuteron behaves like $O(1/N_C)$ as well as the electric quadrupole moment $Q_{\rm D}$ and the coupling constant of the phenomenological local four–nucleon interaction Eq.(3.1). This testifies a self–consistency of our approach. Really, all parameters of the physical deuteron field are of the same order according to the large N_C expansion. This means that the vanishing of the coupling constant of the phenomenological four–nucleon interaction Eq.(3.1) in the limit $N_C \to \infty$ entails the vanishing of all low–energy parameters of the physical deuteron.

4 Electromagnetic properties of the deuteron

The description of the deuteron as a Cooper np–pair changes a little bit the description of the electromagnetic parameters of the deuteron given in Ref.[2]. We do not have more a "bare" deuteron field having magnetic dipole and electric quadrupole moments. Therefore, both the magnetic dipole and electric quadrupole moments have to be described by the one–nucleon loop contributions. For the self–consistent description of the electromagnetic properties of the deuteron we cannot deal with only the baryon current $j_\mu(x)$ given by Eq.(3.2) and have to introduce the tensor current [2]

$$\mathcal{J}^{\mu\nu}(x) = \bar{p^c}(x)\sigma^{\mu\nu}n(x) - \bar{n^c}(x)\sigma^{\mu\nu}p(x), \tag{4.1}$$

where $\sigma^{\mu\nu} = (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)/2$.

The local four–nucleon interaction producing the deuteron as a Cooper np–pair reads now

$$\mathcal{L}_{\rm int}(x) = -\frac{1}{4M^2_{\rm N}}\,J^\dagger_\mu(x)J^\mu(x). \tag{4.2}$$

The baryon current $J^\mu(x)$ is defined by

$$\begin{aligned} J^\mu(x) &= -i\,g_{\rm V}\,[\bar{p^c}(x)\gamma^\mu n(x) - \bar{n^c}(x)\gamma^\mu p(x)] \\ &\quad - \frac{g_{\rm T}}{2M_{\rm N}}\,\partial_\nu[\bar{p^c}(x)\sigma^{\nu\mu}n(x) - \bar{n^c}(x)\sigma^{\nu\mu}p(x)], \end{aligned} \tag{4.3}$$

where $g_{\rm T}$ is a dimensionless phenomenological coupling constant [2]. The contribution of the tensor nucleon current looks like the next–to–leading term in the long–wavelength expansion[2] of an effective low–energy four–nucleon interaction.

The effective Lagrangian of the np–system unstable under creation of the Cooper np–pair with quantum numbers of the deuteron is then defined

$$\begin{aligned} \mathcal{L}^{\rm np}(x) &= \bar{n}(x)\,(i\gamma^\mu\partial_\mu - M_{\rm N})\,n(x) + \bar{p}(x)\,(i\gamma^\mu\partial_\mu - M_{\rm N})\,p(x) \\ &\quad -\frac{1}{4M^2_{\rm N}}\,J^\dagger_\mu(x)J^\mu(x). \end{aligned} \tag{4.4}$$

The linearalized version of the effective Lagrangian Eq.(4.4) containing the interpolating local deuteron field reads

$$\mathcal{L}^{\rm np}(x) \;\to\; \bar{n}(x)\,(i\gamma^\mu\partial_\mu - M_{\rm N})\,n(x) + \bar{p}(x)\,(i\gamma^\mu\partial_\mu - M_{\rm N})\,p(x)$$

[2]Due to proportionality $M_{\rm N} \sim N_C$ this expansion is related to the large N_C expansion.

$$
\begin{aligned}
&+M_0^2\, D^{\dagger}_{\mu}(x)D^{\mu}(x) + g_{\rm V} j^{\dagger}_{\mu}(x)D^{\mu}(x) + g_{\rm V} j^{\mu}(x)D^{\dagger}_{\mu}(x)\\
&+\frac{g_{\rm T}}{M_0} J^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x) + \frac{g_{\rm T}}{M_0} J^{\mu\nu}(x)D^{\dagger}_{\mu\nu}(x), \qquad (4.5)
\end{aligned}
$$

where $M_0 = 2\,M_{\rm N}$, $D^{\mu}(x)$ is a local interpolating field with quantum numbers of the deuteron and $D^{\mu\nu}(x) = \partial^{\mu}D^{\nu}(x) - \partial^{\nu}D^{\mu}(x)$.

The interactions with the tensor current give the contributions only to the divergent part of the effective Lagrangian of the free deuteron field defined now by [2] defined

$$
\begin{aligned}
\mathcal{L}_{\rm eff}(x) = -\frac{1}{2}\Bigg(-\frac{g^2_{\rm V}}{2\pi^2}\,a(a+1) + \frac{g^2_{\rm V} + 6 g_{\rm V} g_{\rm T} + 3 g^2_{\rm T}}{3\pi^2}\,J_2(M_{\rm N})\Bigg)\,D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x)\\
+\Bigg(M^2_0 - \frac{g^2_{\rm V}}{2\pi^2}\,[J_1(M_{\rm N}) + M^2_{\rm N}J_2(M_{\rm N})]\Bigg)\,D^{\dagger}_{\mu}(x)D^{\mu}(x). \qquad (4.6)
\end{aligned}
$$

Due to the relation Eq.(3.16) the effective Lagrangian of the free deuteron field Eq.(4.6) takes the form

$$
\begin{aligned}
\mathcal{L}_{\rm eff}(x) = -\frac{1}{2}\Bigg(1 + \frac{g^2_{\rm V} + 6 g_{\rm V} g_{\rm T} + 3 g^2_{\rm T}}{3\pi^2}\,J_2(M_{\rm N})\Bigg)\,D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x)\\
+\Bigg(M^2_0 - \frac{g^2_{\rm V}}{2\pi^2}\,[J_1(M_{\rm N}) + M^2_{\rm N}J_2(M_{\rm N})]\Bigg)\,D^{\dagger}_{\mu}(x)D^{\mu}(x). \qquad (4.7)
\end{aligned}
$$

After the renormalization of the wave function of the deuteron field we arrived at the effective Lagrangian defined by Eq.(3.19) with the binding energy of the deuteron depending on $g_{\rm V}$ and $g_{\rm T}$ [2]

$$
\begin{aligned}
\varepsilon_{\rm D} &= \frac{17}{48}\,\frac{g^2_{\rm V}}{\pi^2}\,\frac{J_1(M_{\rm N})}{M_{\rm N}}\,\Bigg(1 + \frac{48}{17}\,\frac{g_{\rm T}}{g_{\rm V}} + \frac{24}{17}\,\frac{g^2_{\rm T}}{g^2_{\rm V}}\Bigg) = \\
&= \frac{17}{18}\,Q_{\rm D}\,\Lambda^3_{\rm D}\,\Bigg(1 + \frac{48}{17}\,\frac{g_{\rm T}}{g_{\rm V}} + \frac{24}{17}\,\frac{g^2_{\rm T}}{g^2_{\rm V}}\Bigg), \qquad (4.8)
\end{aligned}
$$

where we have used the relation between divergent integrals Eq.(3.11) and expressed the phenomenological coupling constant $g_{\rm V}$ in terms of the electric quadrupole moment of the deuteron $g^2_{\rm V} = 2\pi^2 Q_{\rm D} M^2_{\rm N}$. In order to make the prediction for the binding energy much more definite we have to know the relation between the phenomenological coupling constants $g_{\rm V}$ and $g_{\rm T}$. For this aim we suggest to consider the electromagnetic properties of the deuteron.

Including the electromagnetic field by a minimal way $\partial_\mu \to \partial_\mu + i\,e\,A_\mu(x)$, where e and $A_\mu(x)$ are the electric charge of the proton and the electromagnetic potential we bring up the linearalized version of the Lagrangian Eq.(4.5) to the form

$$\begin{aligned}
&\mathcal{L}^{\rm np}(x) \to \mathcal{L}^{\rm np}_{\rm ELM}(x) = \\
&= \bar{n}(x)\,(i\gamma^\mu\partial_\mu - M_{\rm N})\,n(x) + \bar{p}(x)\,(i\gamma^\mu\partial_\mu - M_{\rm N})\,p(x) + M^2_0\,D^\dagger_\mu(x)D^\mu(x) \\
&+ g_{\rm V} j^\dagger_\mu(x)D^\mu(x) + g_{\rm V} j^\mu(x)D^\dagger_\mu(x) + \frac{g_{\rm T}}{M_0}J^\dagger_{\mu\nu}D^{\mu\nu}(x) + \frac{g_{\rm T}}{M_0}J^{\mu\nu}D^\dagger_{\mu\nu}(x) \\
&- e\,\bar{p}(x)\gamma^\mu p(x)\,A_\mu(x) - i\,e\,\frac{g_{\rm T}}{M_0}J^\dagger_{\mu\nu}(x)(A^\mu(x)D^\nu(x) - A^\nu(x)D^\mu(x)) \\
&+ i\,e\,\frac{g_{\rm T}}{M_0}J^{\mu\nu}(x)(A_\mu(x)D_\nu(x) - A^\nu(x)D^\mu(x)).
\end{aligned}\tag{4.9}$$

The effective Lagrangian describing both the magnetic dipole and electric quadrupole moments of the deuteron has been evaluated in Ref.[2] and reads

$$\begin{aligned}
\delta\mathcal{L}^{\rm D}_{\rm ELM}(x)_{\rm eff} &= i\,e\,\frac{4ag^2_{\rm T} - bg^2_{\rm V}}{6\,\pi^2}\,D^\dagger_{\mu\nu}(x)\,A^\mu(x)\,D^\nu(x) \\
&- i\,e\,\frac{4ag^2_{\rm T} - bg^2_{\rm V}}{6\,\pi^2}\,D^{\mu\nu}(x)\,A_\mu(x)\,D^\dagger_\nu(x) \\
&+ i\,e\,\frac{g^2_{\rm V}}{6\,\pi^2}\,(2\,b\,+\,3)\,D^\dagger_\mu(x)\,D_\nu(x)\,F^{\mu\nu}(x) \\
&+ i\,e\,(1\,+\,a)\,\frac{2\,g^2_{\rm T}}{3\,\pi^2}\,\frac{1}{M^2_{\rm D}}\,D^\dagger_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,F_\lambda{}^\mu(x)\,,
\end{aligned}\tag{4.10}$$

where $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ is the electromagnetic field strength, a and b are arbitrary parameters related to the ambiguities of the one–nucleon loop diagrams with respect to a shift of virtual nucleon momentum. We consider them as free parameters of the approach [2].

In order to fix parameters it is convenient to write down the total effective Lagrangian of the physical deuteron coupled to an external electromagnetic field

$$\begin{aligned}
\mathcal{L}^{\rm D}_{\rm ELM}(x)_{\rm eff} &= -\frac{1}{2}\,D^\dagger_{\mu\nu}(x)D^{\mu\nu}(x) + M^2_{\rm D}\,D^\dagger_\mu(x)D^\mu(x) \\
&+ i\,e\,\frac{4ag^2_{\rm T} - bg^2_{\rm V}}{6\,\pi^2}\,\frac{1}{6\,\pi^2}\,D^\dagger_{\mu\nu}(x)\,A^\mu(x)\,D^\nu(x) \\
&- i\,e\,\frac{4ag^2_{\rm T} - bg^2_{\rm V}}{6\,\pi^2}\,D^{\mu\nu}(x)\,A_\mu(x)\,D^\dagger_\nu(x)
\end{aligned}$$

$$+ \quad i\,e\,\frac{g^2_{\rm V}}{6\pi^2}\,(2\,b\,+\,3)\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,F^{\mu\nu}(x)$$
$$+ \quad i\,e\,(1\,+\,a)\,\frac{2g^2_{\rm T}}{3\pi^2}\,\frac{1}{M^2_{\rm D}}\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,F_{\lambda}{}^{\mu}(x). \qquad (4.11)$$

Two terms having the structure $D^{\mu\nu}(x)A_{\mu}(x)D^{\dagger}_{\nu}(x)$ and $D^{\dagger}_{\mu\nu}(x)A^{\mu}(x)D^{\nu}(x)$ should describe the interaction of the deuteron with an external electromagnetic field included by a minimal way, whilst the last two terms are responsible for the non–trivial contributions to the magnetic dipole and electric quadrupole moments of the deuteron. In terms of the parameters of the effective interactions Eq.(4.11) the magnetic dipole moment $\mu_{\rm D}$, measured in nuclear magnetons, and the electric quadrupole moment $Q_{\rm D}$, measure in ${\rm fm}^2$, of the deuteron are given by

$$\begin{aligned}\mu_{\rm D} &= \frac{g^2_{\rm T}}{3\pi^2} + (1\,+\,b)\,\frac{g^2_{\rm V}}{4\pi^2},\\ Q_{\rm D} &= \Big[(2\,+\,2\,a)\,\frac{g^2_{\rm T}}{3\pi^2} - (3\,+\,2\,b)\,\frac{g^2_{\rm V}}{6\pi^2}\Big]\,\frac{1}{M^2_{\rm D}}\end{aligned} \qquad (4.12)$$

at the constraint

$$b\,\frac{g^2_{\rm V}}{6\pi^2}\,-\,2\,a\,\frac{g^2_{\rm T}}{3\pi^2} = 1 \qquad (4.13)$$

reducing the first two terms in effective Lagrangian Eq.(4.10) to the standard minimal form which can be obtained from the effective Lagrangian of the free deuteron field by the shift $\partial_{\mu}D_{\nu}(x) \to (\partial_{\mu} + i\,e\,A_{\mu}(x))D_{\nu}(x)$.

By retaining the former relation between the electric quadrupole moment and the coupling constant $g_{\rm V}$, $Q_{\rm D} = 2g^2_{\rm V}/\pi^2 M^2_{\rm D}$ [2], one can show that the experimental values of the magnetic dipole moment of the deuteron $\mu_{\rm D} = 0.857$ and the electric quadrupole moment $Q_{\rm D} = 0.286\,{\rm fm}^2$ can be fitted by the following values of the parameters a and b: $a = -\,0.442$ and $b = -\,4.418$ at $g_{\rm V} = 11.319$ [2]. This gives the relation between the coupling constants $g_{\rm V}$ and $g_{\rm T}$

$$g_{\rm T} = -\,1.662\,g_{\rm V}. \qquad (4.14)$$

Substituting this relation into Eq.(4.8) we can describe the experimental value of the binding energy of the deuteron $\varepsilon_{\rm D} = 2.225\,{\rm MeV}$ at the cut–off $\Lambda_{\rm D} = 115.729\,{\rm MeV}$. The spatial region of virtual nucleon field fluctuations

forming the physical deuteron related to this value of the cut–off $1/\Lambda_{\rm D} \sim \rho_{\rm D} = 1.705\,{\rm fm}$ agrees good with the effective range of the deuteron $\rho_{\rm D} = (1.759 \pm 0.005)\,{\rm fm}$ [6].

The effective Lagrangian of the deuteron field coupled to an external electromagnetic field is given by

$$\begin{aligned}
&{\cal L}^{\rm D}_{\rm ELM}(x)_{\rm eff} = -\frac{1}{2}\,[(\partial_{\mu} - \,i\,e\,A_{\mu}(x))D^{\dagger}_{\nu}(x) - (\partial_{\nu} - \,i\,e\,A_{\nu}(x))D^{\dagger}_{\mu}(x)]\\
&\times [(\partial^{\mu} + \,i\,e\,A^{\mu}(x))D^{\nu}(x) - (\partial^{\nu} + \,i\,e\,A^{\nu}(x))D^{\mu}(x)] + M^2_{\rm D}\,D^{\dagger}_{\mu}(x)D^{\mu}(x)\\
&-i\,e\,\bar{\mu}_{\rm D}\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,F^{\mu\nu}(x) + \,i\,e\,\bar{Q}_{\rm D}\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,F_{\lambda}{}^{\mu}(x),
\end{aligned}\tag{4.15}$$

where $\bar{\mu}_{\rm D} = 14.682\,\mu_{\rm D}$ and $\bar{Q}_{\rm D} = 0.514\,Q_{\rm D}$. The term of order $O(e^2)$ can be also derived in the RFMD by using shift ambiguities of one–nucleon loop diagrams. This term is required by the electromagnetic gauge invariance of the effective Lagrangian of the deuteron field, but it does not affect on the electromagnetic parameters of the deuteron which are of order $O(e)$.

Conclusion

Unlike the first formulation of the RFMD given in Refs. [1,2], where we have stated that the RFMD *is far from being induced by the dynamics of QCD and seems like an old–fashion approach* [2], in this paper we have shown that the RFMD is in complete agreement with QCD and can be formulated from the first principles of QCD. The RFMD describes low–energy nuclear forces in the nuclear phase of QCD in terms of one–nucleon loop exchanges. One–nucleon loop exchanges provide a minimal way of the transfer of nucleon flavours from an initial to a final nuclear state and allow to take into account contributions of nucleon–loop anomalies. These anomalies are related to high–energy fluctuations of virtual nucleon fields, i.e. the $N\bar{N}$ fluctuations, and fully determined by one–nucleon loop diagrams [33–35]. The dominance of contributions of one–nucleon loop anomalies to effective Lagrangians describing low–energy interactions of the deuteron coupled to itself and other particles is justified in the large N_C expansion in QCD at $N_C \to \infty$. It is well–known that anomalies of quark–loop diagrams play an important role for the correct description of strong low–energy interactions of low–lying hadrons. We argue an important role of nucleon–loop anomalies for the correct description of low–energy nuclear forces in the nuclear physics.

It is interesting that nucleon–loop anomalies can be interpreted as non–trivial contributions of the non–perturbative quantum vacuum – the nucleon Dirac sea [17]. In nuclear physics the influence of the nucleon Dirac sea for low–energy properties of finite nuclei has been analysed within quantum field theoretic approaches in the one–nucleon loop approximation [16]. Unfortunately, in these approaches contributions of one–nucleon loop anomalies have been taken into account. The RFMD allows to fill this blank.

For the derivation of the RFMD from the first principles of QCD we distinguish three non–perturbative phases of QCD: 1) the low–energy quark–gluon phase (low–energy QCD), 2) the hadronic phase and 3) the nuclear phase. Skipping over the intermediate low–energy quark–gluon phase by means of the integration over high– and low–energy quark and gluon fluctuations one arrives at the hadronic phase of QCD containing only local hadron fields with quantum numbers of mesons and baryons coupled at energies below the SBχS scale $\Lambda_\chi \simeq 1\,\mathrm{GeV}$. The couplings of low–lying mesons with masses less than the SBχS scale to low–lying octet and decuplet of baryons can be described by Effective Chiral Lagrangians with chiral $U(3)\times U(3)$ symmetry.

Integrating in the hadronic phase of QCD over heavy hadron degrees of freedom with masses exceeding the SBχS scale one arrives at the nuclear phase of QCD which characterizes itself by the appearance of bound nucleon states – nuclei. At low energies the result of integration over heavy hadron degrees of freedom can be represented in the form of phenomenological local many–nucleon interactions. Some of these interactions are responsible for creation of many–nucleon collective excitations which acquire the properties of observed nuclei through nucleon–loop and low–lying meson exchanges. This effective field theory describes nuclei and processes of their low–energy interactions by considering nuclei as elementary particles [28,29].

Following this scenario of the description of nuclei and their low–energy interactions from the first principles of QCD the deuteron should be produced in the nuclear phase of QCD by a phenomenological local four–nucleon interaction as the Cooper np–pair with quantum numbers of the deuteron. The properties of the physical deuteron, i.e. the binding energy, the electric quadrupole moment and so, the Cooper np–pair acquires through one–nucleon loop exchanges. As has been shown the main part of the kinetic term of the effective Lagrangian of the free physical deuteron field is induced by the contribution of high–energy (short–distance) fluctuations of virtual nucleon fields related to the anomaly of the one–nucleon loop diagram with two vector vertices – the VV–diagram.

In turn, the magnetic dipole $\mu_{\rm D}$ and electric quadrupole moments of the physical deuteron $Q_{\rm D}$ are fully determined by high–energy (short–distance) fluctuations of virtual nucleon fields related to the anomalous contributions of the one–nucleon loop diagrams with three vector vertices (the γVV–diagram) [2]. Thus, high–energy (short–distance) fluctuations of nucleon fields related to anomalies of one–nucleon loop diagrams play a dominant role for the formation of the physical deuteron from the Cooper np–pair.

As regards low–energy (long–distance) fluctuations of virtual nucleon fields they give a significant contribution only to the binding energy of the deuteron $\varepsilon_{\rm D}$. The strength of low–energy (long–distance) fluctuations of virtual nucleon fields is restricted by the cut–off $\Lambda_{\rm D} = 115.729\,{\rm MeV}$. The spatial region of virtual nucleon field fluctuations forming the physical deuteron related to this value of the cut–off $1/\Lambda_{\rm D} \sim \rho_{\rm D} = 1.705\,{\rm fm}$ agrees good with the effective range of the deuteron $\rho_{\rm D} = (1.759 \pm 0.005)\,{\rm fm}$ [6].

It is well–known that in the potential model approach to the description of the deuteron the electric quadrupole moment of the deuteron $Q_{\rm D}$ is caused by nuclear tensor forces which play an important role for the existence of the deuteron as a bound np–state.

The proportionality of the coupling constant of the phenomenological local four–nucleon interaction Eq.(3.3), responsible for creation of the Cooper np–pair with quantum numbers of the deuteron, and the binding energy of the deuteron $\varepsilon_{\rm D}$ Eq.(3.20) to the electric quadrupole moment $Q_{\rm D}$ testifies an important role of nuclear tensor forces for the formation of the deuteron in the RFMD.

To the evaluation of one–nucleon loop diagrams defining effective Lagrangians describing processes of low–energy interactions of the deuteron coupled to itself and other particles we apply expansions in powers of the momenta of interacting particles and keep only leading terms of the expansions [1–5]. This approximation can be justified in the large N_C expansion. Indeed, in QCD with the $SU(N_C)$ gauge group at $N_C \to \infty$ the nucleon mass is proportional to the number of quark colours [19]: $M_{\rm N} \sim N_C$. Since for the derivation of effective Lagrangians describing the deuteron and amplitudes of low–energy nuclear processes all external momenta of interacting particles should be kept off–mass shell, the masses of virtual nucleon fields are larger compared with the external momenta. An expansion of one–nucleon loop diagrams in powers of $1/M_{\rm N}$ giving an external momentum expansion corresponds to the expansion in powers of $1/N_C$. In this case the leading order in the large N_C expansion gives the leading order contributions in the expansion

in powers of external momenta of interacting particles. We should emphasize that anomalous contributions of one–nucleon loop diagrams are determined by the least powers in external momentum expansions. Thereby, the dominance of contributions of nucleon–loop anomalies to effective Lagrangians describing low–energy nuclear processes in the RFMD is fully supported by the large N_C expansion. The accuracy of this approximation is rather high. Indeed, the real parameter of the expansion of one–nucleon loop diagrams is $1/M^2_{\rm N} \sim 1/N^2_C$ but not $1/M_{\rm N} \sim 1/N_C$. Thereby, next–to–leading corrections should be of order $O(1/N^2_C)$.

The inclusion of the interaction of the deuteron field with the tensor nucleon current Eq.(4.1) has given a possibility of the self-consistent description of the electromagnetic properties of the deuteron, the magnetic dipole moment $\mu_{\rm D}$ and the electric quadrupole moment $Q_{\rm D}$, in terms of effective interactions of the Corben–Schwinger and Aronson kinds induced by one–nucleon loop diagrams [2]. By fitting the experimental values of the magnetic dipole moment $\mu_{\rm D} = 0.857\,\mu_{\rm N}$, where $\mu_{\rm N} = e/2M_{\rm N}$ is a nuclear magneton, and the electric quadrupole moment $Q_{\rm D} = 0.286\,{\rm fm}^2$ supplemented by the requirement of the electromagnetic gauge invariance of the effective Lagrangian of the deuteron field coupled to an external electromagnetic field [2] we have got the relation between the coupling constants $g_{\rm V}$ and $g_{\rm T}$: $g_{\rm T} = -\,1.662\,g_{\rm V}$. Due to this relation the experimental value of the binding energy of the deuteron can be described by the cut–off $\Lambda_{\rm D} = 115.729\,{\rm MeV}$. This corresponds to the spatial region of virtual nucleon field fluctuations forming the physical deuteron $1/\Lambda_{\rm D} \sim \rho_{\rm D} = 1.705\,{\rm fm}$ agreeing good with the effective range of the deuteron $\rho_{\rm D} = (1.759 \pm 0.005)\,{\rm fm}$ [6]. As has been stated in Ref.[2] the contributions of the tensor nucleon current Eq.(4.1) to the amplitudes of low–energy nuclear processes of astrophysical interest such as the neutron–proton radiative capture $\rm n + p \to D + \gamma$ for thermal neutrons and the solar proton burning $\rm p + p \to D + e^+ + \nu_e$ can be neglected. The same statement is valid for the disintegration of the deuteron by anti–neutrinos and neutrinos induced by charged $\bar{\nu}_{\rm e} + {\rm D} \to {\rm e}^+ + {\rm n} + {\rm n}$, $\nu_{\rm e} + {\rm D} \to {\rm e}^- + {\rm p} + {\rm p}$ and neutral $\bar{\nu}_{\rm e}(\nu_{\rm e}) + {\rm D} \to \bar{\nu}_{\rm e}(\nu_{\rm e}) + {\rm n} + {\rm p}$ weak currents, and the pep process, ${\rm p} + {\rm e}^- + {\rm p} \to {\rm D} + \nu_{\rm e}$.

The quantum field theoretic scenario to treating nuclei as many–nucleon collective excitations induced by phenomenological local many–nucleon interactions allows a plain extension of the RFMD by the inclusion of light nuclei ^{3}He, ^{3}H and ^{4}He as three– and four–nucleon collective excitations. The binding energies and other low–energy parameters of these excitations

should be defined through nucleon–loop and low–lying meson exchanges. The extension of the RFMD by the inclusion of ^{3}He, ^{3}H and ^{4}He would give a possibility to continue investigations of the reactions of the p–p chain [36] started with the reaction $\mathrm{p} + \mathrm{p} \to \mathrm{D} + \mathrm{e}^+ + \nu_{\mathrm{e}}$ and to apply the extended version of the RFMD to the description of the reactions $\mathrm{p} + \mathrm{D} \to {}^3\mathrm{He} + \gamma$, $\mathrm{p} + {}^3\mathrm{He} \to {}^4\mathrm{He} + \mathrm{e}^+ + \nu_{\mathrm{e}}$ and so on.

We would like to emphasize that Chiral perturbation theory can be naturally incorporated into this effective field theoretic approach to physics of low–energy interactions of nuclei in terms of Effective Chiral Lagrangians with chiral $U(3) \times U(3)$ symmetry describing low–lying baryons and mesons interacting at low energies [37]. Unfortunately, the discussion of the inclusion of Chiral perturbation theory into the RFMD and the description of low–energy nuclear processes in the RFMD goes beyond the scope of this paper. Therefore, we relegate readers to Refs.[37,38], where these problems and a comparison of the RFMD with other approaches have been discussed in details.

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